

Class IX Session 2023-24
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section A

1. The value of $2.\overline{45} + 0.\overline{36}$ is [1]
 - a) $\frac{31}{11}$
 - b) $\frac{24}{11}$
 - c) $\frac{67}{33}$
 - d) $\frac{167}{110}$
2. The equation $x = 7$ in two variables can be written as [1]
 - a) $1.x + 1.y = 7$
 - b) $1.x + 0.y = 7$
 - c) $0.x + 1.y = 7$
 - d) $0.x + 0.y = 7$
3. The point which lies on y-axis at a distance of 6 units in the positive direction of y-axis is [1]
 - a) (-6, 0)
 - b) (0, -6)
 - c) (6, 0)
 - d) (0, 6)
4. In a histogram, which of the following is proportional to the frequency of the corresponding class? [1]
 - a) Width of the rectangle
 - b) Length of the rectangle
 - c) Perimeter of the rectangle
 - d) Area of the rectangle
5. How many linear equations in 'x' and 'y' can be satisfied by $x = 1, y = 2$? [1]
 - a) Infinitely many
 - b) Two
 - c) Only one
 - d) Three
6. The number of dimensions, a surface has [1]

a) 2

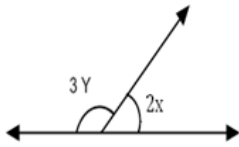
b) 1

c) 0

d) 3

7. In the given figure $x = 30^\circ$, the value of Y is

[1]



a) 45°

b) 40°

c) 10°

d) 36°

8. If APB and CQD are 2 parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form, square only if

[1]

a) Diagonals of ABCD are equal

b) ABCD is a Rhombus

c) None of these

d) Diagonals of ABCD are unequal

9. Degree of the polynomial $2x^4 + 3x^3 - 5x^2 + 9x + 1$ is

[1]

a) 3

b) 1

c) 2

d) 4

10. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹160. A linear equation in two variables to represent the above data is

[1]

a) $x - 2y = 160$

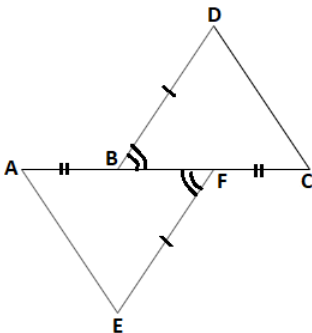
b) $2x + y = 160$

c) $x + y = 160$

d) $2x - y = 160$

11. In the adjoining figure, $AB = FC$, $EF = BD$ and $\angle AFE = \angle CBD$. Then the rule by which $\triangle AFE \cong \triangle CBD$

[1]



a) SSS

b) AAS

c) ASA

d) SAS

12. The figure formed by joining the mid-points of the adjacent sides of a rhombus is a

[1]

a) trapezium

b) rectangle

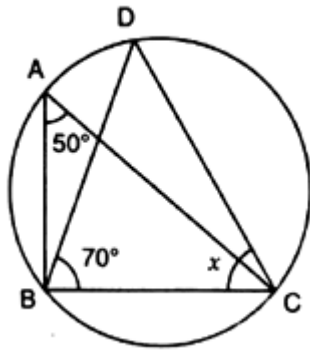
c) square

d) none of these

13. In the given figure, M, A, B and N are points on a circle having centre O. AN and MB cut at Y. If $\angle NYB = 50^\circ$ and $\angle YNB = 20^\circ$, then reflex $\angle MON$ is equal to

[1]

22. If O is the centre of the circle, find the value of x in the given figure: [2]



23. Find the volume, curved surface area and the total surface area of a hemisphere of diameter 7 cm. [2]
 24. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA \cong arc PYB. [2]

OR

Two chords PQ and RS of a circle are parallel to each other and AB is the perpendicular bisector of PQ. Without using any construction, prove that AB bisects RS.

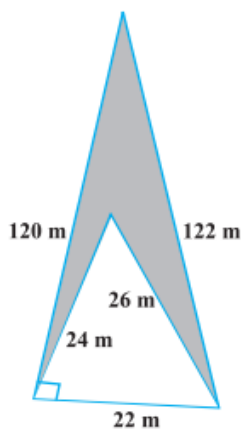
25. How many solution(s) of the equation $3x + 2 = 2x - 3$ are there on the : [2]
 i. Number line?
 ii. Cartesian plane?

OR

Solve the equation for x: $5(4x + 3) = 3(x - 2)$

Section C

26. Represent $\sqrt{9.3}$ on the number line. [3]
 27. Factorize: $a^2x^2 + (ax^2 + 1)x + a$ [3]
 28. Calculate the area of the shaded region in Fig. [3]



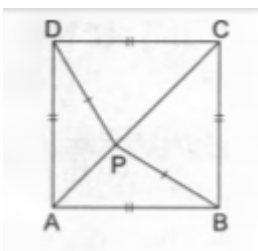
OR

A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's Formula. If its perimeter is 180 cm,

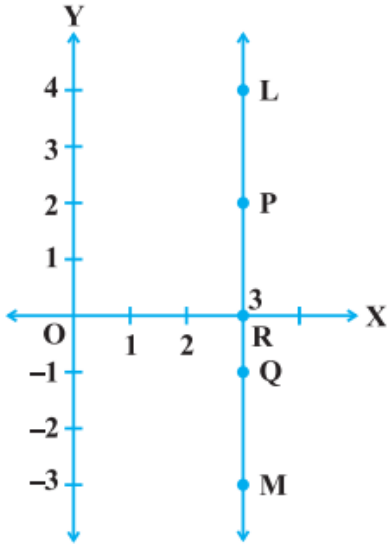
29. Write linear equation $3x + 2y = 18$ in the form of $ax + by + c = 0$. Also write the values of a, b and c. Are (4, 3) and (1, 2) solution of this equation? [3]
 30. Prove that the angle between internal bisector of one base angle and the external bisector of the other base angle of a triangle is equal to one-half of the vertical angle. [3]

OR

In the given figure, ABCD is a square and P is a point inside it such that $PB = PD$. Prove that CPA is a straight line.



31. In Figure, LM is a line parallel to the y-axis at a distance of 3 units. [3]



- What are the coordinates of the points P, R and Q?
- What is the difference between the abscissa of the points L and M?

Section D

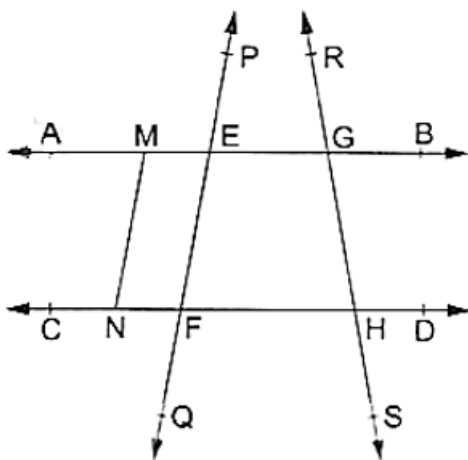
32. If $a = 3 - 2\sqrt{2}$, find the value of $a^2 - \frac{1}{a^2}$. [5]

OR

If $p = \frac{3-\sqrt{5}}{3+\sqrt{5}}$ and $q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$, find the value of $p^2 + q^2$.

33. In the adjoining figure, name: [5]

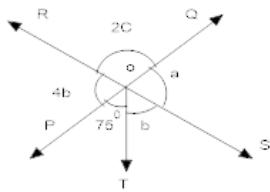
- Six points
- Five line segments
- Four rays
- Four lines
- Four collinear points



34. If it is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$. [5]

OR

In fig two straight lines PQ and RS intersect each other at O, if $\angle POT = 75^\circ$ Find the values of a, b and c



35. Draw a histogram with frequency polygon for the following data: [5]

class interval	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54
frequency	5	15	23	20	10	7

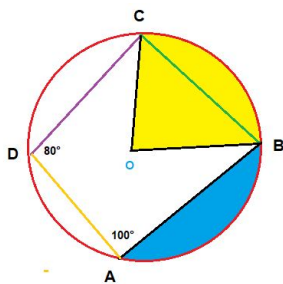
Section E

36. Read the text carefully and answer the questions: [4]

There was a circular park in Defence colony at Delhi. For fencing purpose poles A, B, C and D were installed at the circumference of the park.

Ram tied wires From A to B, B to C and C to D, and he managed to measure the $\angle A = 100^\circ$ and $\angle D = 80^\circ$

Point O in the middle of the park is the center of the circle.



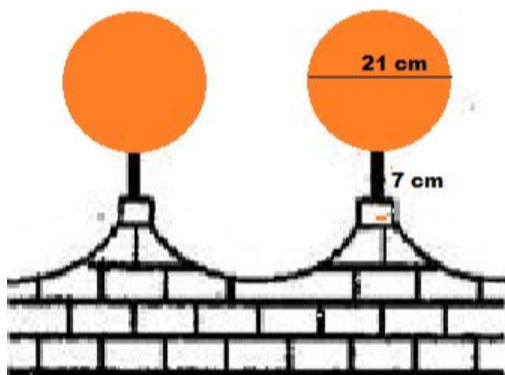
- (i) Name the quadrilateral ABCD.
- (ii) What is the value of $\angle C$?
- (iii) What is the value of $\angle B$.

OR

Write any three properties of cyclic quadrilateral?

37. Read the text carefully and answer the questions: [4]

The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in figure. 25 such spheres are used for this purpose and are to be painted silver. Each support is a cylinder and is to be painted black.



- (i) what will be the total surface area of the spheres all around the wall?
- (ii) Find the cost of orange paint required if this paint costs 20 paise per cm^2 .
- (iii) How much orange paint in liters is required for painting the supports if the paint required is 3 ml per cm^2 ?

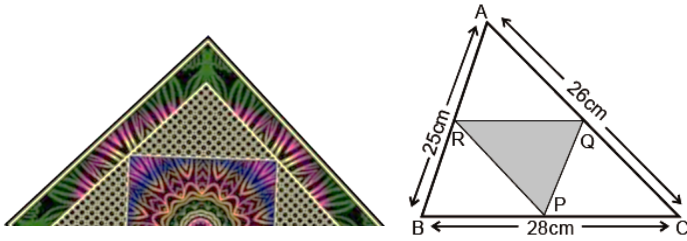
OR

What will be the volume of total spheres all around the wall?

38. Read the text carefully and answer the questions:

[4]

There is a Diwali celebration in the DPS school Janakpuri New Delhi. Girls are asked to prepare Rangoli in a triangular shape. They made a rangoli in the shape of triangle ABC. Dimensions of $\triangle ABC$ are 26 cm, 28 cm, 25 cm.



- (i) In fig R and Q are mid-points of AB and AC respectively. Find the length of RQ.
- (ii) Find the length of Garland which is to be placed along the side of $\triangle PQR$.
- (iii) R, P and Q are the mid-points of AB, BC, and AC respectively. Then find the relation between area of $\triangle PQR$ and area of $\triangle ABC$.

OR

R, P, Q are the mid-points of corresponding sides AB, BC, CA in $\triangle ABC$, then name the figure so obtained BPQR.

Solution

Section A

1. (a) $\frac{31}{11}$

Explanation: $2.\overline{45} + 0.\overline{36}$

$$= 2.\overline{81}$$

$$= \frac{279}{99}$$

$$= \frac{31}{11}$$

2.

(b) $1.x + 0.y = 7$

Explanation: The equation $x = 7$ in two variables can be written as exactly $1.x + 0.y = 7$ because it contains two variables x and y and the coefficient of y is zero as there is no term containing y in the equation $x = 7$.

3.

(d) (0, 6)

Explanation: Since it lies on the y -axis, its abscissa x will be zero. Thus, the point will be (0, 6).

4.

(b) Length of the rectangle

Explanation: In a histogram, each rectangle is drawn, where the width is equivalent to the class interval and the height is equivalent to the frequency of the class.

5. (a) Infinitely many

Explanation: There are many linear equations in ' x ' and ' y ' that can be satisfied by $x = 1, y = 2$ for example

$$x + y = 3 \quad x - y = -1$$

$$2x + y = 4$$

and so on there are infinite numbers of examples.

6. (a) 2

Explanation: The surface is that which has length and breadth. (1 dimension + 1 dimension = 2 dimension)

7.

(b) 40°

Explanation: In the given figure we have

$$3Y + 2X = 180^\circ \text{ (Linear - Pair)}$$

$$X = 30^\circ$$

$$3Y + 2 \times 30^\circ = 180^\circ$$

$$3Y + 60^\circ = 180^\circ$$

$$3Y = 180^\circ - 60^\circ$$

$$3Y = 120^\circ$$

$$Y = \frac{120^\circ}{3}$$

$$Y = 40^\circ$$

8. (a) Diagonals of ABCD are equal

Explanation: The diagonals of a square bisect its angles. Opposite sides of a square are both parallel and equal in length. All four angles of a square are equal.

9.

(d) 4

Explanation: The highest power of the variable is 4. So, the degree of the polynomial is 4.



10.

(b) $2x + y = 160$

Explanation: Let the cost of apples be ₹x per Kg and cost of grapes be ₹y per Kg. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹160.

So the equation will be

$2x + y = 160$

11.

(d) SAS

Explanation: In $\triangle DBC$ and $\triangle AEF$, we have

$AB = FC$ (given) by adding BF on both sides

$AF = CB$

$\angle AFE = \angle CBD$ (given)

$EF = BD$ (given)

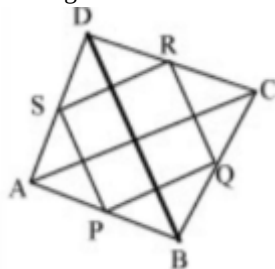
Hence, $\triangle AFE \cong \triangle CBD$ by SAS as the corresponding sides and their included angles are equal.

12.

(b) rectangle

Explanation:

rectangle



Let ABCD be a rhombus and P, Q, R and S be the mid-points of sides AB, BC, CD and DA respectively.

In $\triangle ABD$ and $\triangle BDC$ we have

$SP \parallel BD$ and $SP = \frac{1}{2}BD$ (1) [By mid-point theorem]

$RQ \parallel BD$ and $RQ = \frac{1}{2}BD$ (2) [By mid-point theorem]

From (1) and (2) we get,

$SP \parallel RQ$

PQRS is a parallelogram

As diagonals of a rhombus bisect each other at right angles.

$\therefore AC \perp BD$

Since, $SP \parallel BD$, $PQ \parallel AC$ and $AC \perp BD$

$\therefore SP \perp PQ$

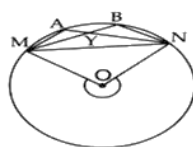
$\therefore \angle QPS = 90^\circ$

\therefore PQRS is a rectangle.

13.

(d) 220°

Explanation:



In triangle NYB,

$\angle N + \angle Y + \angle B = 180^\circ$

$\Rightarrow \angle B = 180^\circ - 50^\circ - 20^\circ = 110^\circ$

Complete the cyclic quadrilateral, MBNX, where X being any point on the circumference in the major segment, we have:-

$\angle MXN = 180^\circ - 110^\circ = 70^\circ$

So, minor $\angle MON = 70^\circ \times 2 = 140^\circ$

Hence, reflex $\angle MON = 360^\circ - 140^\circ = 220^\circ$

14.

(c) 10

Explanation: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\Rightarrow \frac{(\sqrt{3}+\sqrt{2})^2 + (\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$\Rightarrow \frac{(3+2+2\sqrt{6}) + (3+2-2\sqrt{6})}{3-2}$$

$$\Rightarrow 10$$

15. (a) $x + y = 0$

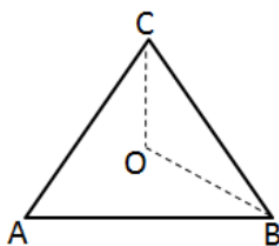
Explanation: Linear equation has solutions $(-2, 2)$, $(0, 0)$ and $(2, -2)$, then the equation will be $x + y = 0$

As all the given three points satisfy the given equation

16.

(d) 120°

Explanation:



O is point where bisectors of $\angle C$ & $\angle B$ meet.

$$\angle A + \angle B + \angle C = 180^\circ$$

$$60^\circ + 80^\circ + \angle C = 180^\circ$$

$$\angle C = 40^\circ$$

$$\frac{\angle C}{2} = 20^\circ$$

$$\frac{\angle C}{2} = 20^\circ = \angle BCO \dots(i)$$

$$\frac{\angle B}{2} = \frac{80^\circ}{2} = 40^\circ = \angle OBC \dots(ii)$$

In $\triangle BOC$

$$\angle BCO + \angle OBC + \angle BOC = 180^\circ$$

From (i) and (ii)

$$20^\circ + 40^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 180^\circ - 60^\circ = 120^\circ$$

17.

(b) $10x$

Explanation: Now, $(25x^2 - 1) + (1 + 5x)^2$

$$= 25x^2 - 1 + 1 + 25x^2 + 10x \text{ [using identity, } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= 50x^2 + 10x = 10x(5x + 1)$$

Hence, one of the factor of given polynomial is $10x$.

18.

(d) $2 : 1$

Explanation: CSA of cone = CSA of cylinder

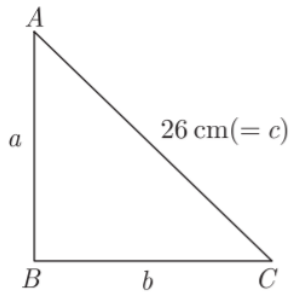
$$\pi r l = 2\pi r h$$

$$l = 2h$$

$$l : h = 2 : 1$$

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation:



$$a + b + c = 60$$

$$a + b + 26 = 60$$

$$a + b = 34 \dots(i)$$

$$\text{Now, } 26^2 = a^2 + b^2 \dots(ii)$$

Squaring (1) both sides, we get

$$(a + b)^2 = (34)^2$$

$$a^2 + b^2 + 2ab = 34 \times 34$$

$$(26)^2 + 2ab = 1156 \text{ [From (ii)]}$$

$$2ab = 1156 - 676$$

$$2ab = 480$$

$$ab = 240 \dots(iii)$$

$$\text{Now, } a + \frac{240}{a} = 34 \text{ [From (i) and (iii)]}$$

$$a^2 - 24a - 10a + 240 = 0$$

$$a(a - 24) - 10(a - 24) = 0$$

$$a = 10, 24$$

Now, other sides are 10 cm and 24 cm

$$s = \frac{26+10+24}{2} = 30 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{30(30 - 26)(30 - 10)(30 - 24)}$$

$$= \sqrt{30 \times 4 \times 20 \times 6} = 120 \text{ cm}^2$$

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. Height of the equilateral triangle = 9 cm

Thus, we have:

$$\text{Height} = \frac{\sqrt{3}}{2} \times \text{Side}$$

$$\Rightarrow 9 = \frac{\sqrt{3}}{2} \times \text{Side}$$

$$\Rightarrow \text{Side} = \frac{18}{\sqrt{3}} = \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 6\sqrt{3} \text{ cm}$$

Also,

$$\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

$$= \frac{\sqrt{3}}{4} \times (6\sqrt{3})^2$$

$$= \frac{108}{4} \sqrt{3}$$

$$= 27\sqrt{3} = 27 \times 1.732$$

$$\text{Area of an equilateral triangle} = 46.76 \text{ cm}^2$$

22. We have, $\angle BAC = 50^\circ$

$$\angle DBC = 70^\circ$$

Therefore, $\angle BDC = \angle BAC = 50^\circ$... (Angles on same segment)

In triangle BDC, by angle sum property

$$\angle BDC + \angle BCD + \angle DBC = 180^\circ$$

$$50^\circ + x + 70^\circ = 180^\circ$$

$$120^\circ + x = 180^\circ$$

$$x = 60^\circ$$

23. Given that radius of the hemisphere, $r = 3.5$ cm.

$$\text{Therefore, Volume of the hemisphere} = \left(\frac{2}{3}\pi r^3\right) \text{ cm}^3$$

$$= \left(\frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}\right) \text{ cm}^3$$

$$= \frac{539}{6} \text{ cm}^3 = 89.93 \text{ cm}^3$$

$$\text{Curved surface area of the hemisphere} = (2\pi r^2) \text{ cm}^2$$

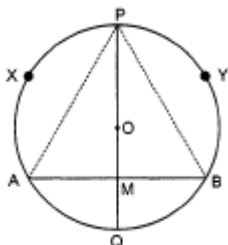
$$= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{ cm}^2 = 77 \text{ cm}^2$$

$$\text{Total surface area of the hemisphere} = (3\pi r^2) \text{ cm}^2$$

$$= \left(3 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{ cm}^2 = \frac{231}{2} \text{ cm}^2 = 115.5 \text{ cm}^2$$

24. Give: In the figure PQ is perpendicular bisector of chord AB

To prove : arc PXA = arc PYB



Construction: Join AP and BP.

Proof: In $\triangle APM$ and $\triangle BPM$

$AM = MB$ (given)

$\angle PMA = \angle PMB$ (90° each)

$PM = PM$ (Common)

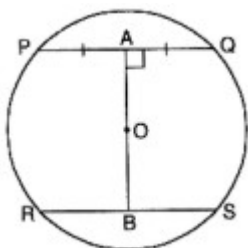
$\triangle APM \cong \triangle BPM$ (SAS)

$PA = PB$ (CPCT)

Hence, arc PXA \cong arc PYB. (As the arc of equal chords are equal)

OR

Given: Two chords PQ and RS of a circle are parallel to each other and AB is the perpendicular bisector of PQ.



To prove: AB bisects RS

Proof: \because AB is the perpendicular bisector of PQ

\therefore AB passes through the centre O [\because The perpendicular bisector of a chord of a circle passes through the centre]

$\therefore PQ \parallel RS$

$\therefore AB \perp RS$

\therefore AB passes through the centre

\therefore AB bisects RS [\because The perpendicular drawn from the centre of a circle bisects the chord]

25. According to the question, given equation is $3x + 2 = 2x - 3$

$$\text{i. } 3x + 2 = 2x - 3$$

$$\Rightarrow 3x - 2x = -3 - 2$$

$$\Rightarrow x = -5$$

So, on a number line there is only one solution which is $x = -5$.

ii. In a Cartesian plane there are infinitely many solutions.

OR

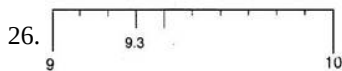
According to the question, given equation is $5(4x + 3) = 3(x-2)$.

$$\Rightarrow 20x + 15 = 3x - 6$$

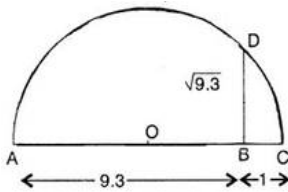
$$\Rightarrow 20x - 3x = -6-15$$

$$\Rightarrow 17x = -21 \Rightarrow x = \frac{-21}{17}$$

Section C



The distance 9.3 from a fixed point A on a given line to obtain a point B such that $AB = 9.3$ units. From B mark a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semi-circle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Then $BD = \sqrt{9.3}$.



27. The given expression may be rewritten as,

$$a^2x^2 + ax^3 + x + a$$

Taking common ax^2 in $(a^2x^2 + ax^3)$ and 1 in $(x + a)$

$$= ax^2(a + x) + 1(x + a)$$

$$= ax^2(a + x) + 1(a + x)$$

Taking $(a + x)$ common in both the terms

$$(a + x)(ax^2 + 1)$$

$$\therefore a^2x^2 + (ax^2 + 1)x + a = (a + x)(ax^2 + 1)$$

28. For the triangle having the sides 122 m, 120 m and 22 m:

$$s = \frac{122+120+22}{2} = 132$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{132(132-122)(132-120)(132-22)}$$

$$= \sqrt{132 \times 10 \times 12 \times 110}$$

$$= 1320 \text{ m}^2$$

For the triangle having the side 22m, 24m and 26m:

$$s = \frac{22+24+26}{2} = 36$$

$$\text{Area of the triangle} = \sqrt{36(36-22)(36-24)(36-26)}$$

$$= \sqrt{36 \times 14 \times 12 \times 10}$$

$$= 24\sqrt{105}$$

$$= 24 \times 10.25 \text{ m}^2 \text{ (approx.)}$$

$$= 246 \text{ cm}^2$$

Therefore, the area of the shaded portion.

$$= \text{Area of larger triangle} - \text{Area of smaller (shaded) triangle.}$$

$$= (1320 - 246) \text{ m}^2$$

$$= 1074 \text{ m}^2$$

OR

'a' = a, 'b' = a and 'c' = a.

$$\therefore s = \frac{a'+b'+c'}{2} = \frac{a+a+a}{2} = \frac{3a}{2}$$

\therefore Area of the signal board

$$= \sqrt{s(s-a')(s-b')(s-c')}$$

$$= \sqrt{\frac{3a}{2} \left(\frac{3a}{2} - a\right) \left(\frac{3a}{2} - a\right) \left(\frac{3a}{2} - a\right)}$$

$$= \sqrt{\frac{3a}{2} \left(\frac{a}{2}\right) \left(\frac{a}{2}\right) \left(\frac{a}{2}\right)} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4} a^2$$

Perimeter = 180 cm

$$'a' + 'b' + 'c' = 180$$

$$\therefore a + a + a = 180$$

$$\therefore 3a = 180$$

$$\therefore a = 60 \text{ cm.}$$

$$\begin{aligned} \therefore \text{Area of the signal board} &= \frac{\sqrt{3}}{4} a^2 \\ &= \frac{\sqrt{3}}{4} (60)^2 = 900\sqrt{3} \text{ cm}^2 \end{aligned}$$

Alternatively,

$$s = \frac{3a}{2} = \frac{3}{2}(60) = 90 \text{ cm}$$

Area of the signal board

$$\begin{aligned} &= \sqrt{s(s-a')(s-b')(s-c')} \\ &= \sqrt{90(90-60)(90-60)(90-60)} \\ &= \sqrt{90(30)(30)(30)} \\ &= 900\sqrt{3} \text{ cm}^2 \end{aligned}$$

29. We have the equation as $3x + 2y = 18$

In standard form

$$3x + 2y - 18 = 0$$

$$\text{Or } 3x + 2y + (-18) = 0$$

But standard linear equation is

$$ax + by + c = 0$$

On comparison we get, $a = 3$, $b = 2$, $c = -18$

If $(4, 3)$ lie on the line, i.e., solution of the equation LHS = RHS

$$\therefore 3(4) + 2(3) = 18$$

$$12 + 6 = 18$$

$$18 = 18$$

As LHS = RHS, Hence $(4, 3)$ is the solution of given equation.

Again for $(1, 2)$

$$3x + 2y = 18$$

$$\therefore 3(1) + 2(2) = 18$$

$$3 + 4 = 18$$

$$7 = 18$$

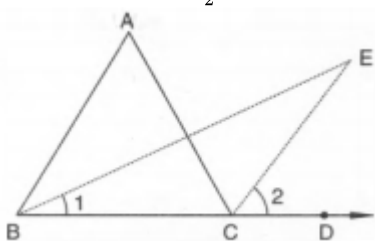
$$\text{LHS} \neq \text{RHS}$$

Hence $(1, 2)$ is not the solution of given equation.

Therefore $(4, 3)$ is the point where the equation of the line $3x + 2y = 18$ passes through where as the line for the equation $3x + 2y = 18$ does not pass through the point $(1, 2)$.

30. **GIVEN** A $\triangle ABC$ with base BC. The internal bisector of $\angle B$ and the external bisector of ext. $\angle ACD$ meet at E.

TO PROVE $\angle E = \frac{1}{2} \angle A$



PROOF Using exterior angle theorem in $\triangle ABC$, we obtain

$$\text{ext. } \angle ACD = \angle A + \angle B$$

$$\Rightarrow \frac{1}{2} \text{ext. } \angle ACD = \frac{1}{2} \angle A + \frac{1}{2} \angle B$$

$$\Rightarrow \angle 2 = \angle 1 + \frac{1}{2} \angle A \dots (i) \quad [\because \text{BE and CE are bisectors of } \angle B \text{ and } \angle ACD \text{ respectively } \therefore \angle B = 2\angle 1 \text{ and ext. } \angle ACD = 2\angle 2]$$

Using exterior angle theorem in $\triangle BCE$, we obtain

$$\text{ext. } \angle ECD = \angle 1 + \angle E$$

$$\Rightarrow \angle 2 = \angle 1 + \angle E \dots (ii)$$

From (i) and (ii), we get

$$\Rightarrow \angle 1 + \frac{1}{2} \angle A = \angle 1 + \angle E$$

$$\Rightarrow \frac{1}{2} \angle A = \angle E$$

$$\Rightarrow \angle E = \frac{1}{2} \angle A$$

OR

CPA is a straight line

In $\triangle APD$ and $\triangle APB$

$$DA = AB$$

$$AP = AP$$

$$PB = PD$$

Thus by SSS {side- side- side} criterion of congruence, we have

$$\triangle APD \cong \triangle APB$$

Now consider the triangles, $\triangle CPD$ and $\triangle CPB$

$$CD = CB$$

$$CP = CP$$

$$PB = PD$$

Thus by side side side criterion of congruence, we have

$$\triangle CPD \cong \triangle CPB.$$

$$\angle APD + \angle CPD = \angle APB + \angle CPB$$

$$\Rightarrow \angle APB + \angle CPB = 360^\circ - (\angle APD + \angle CPD)$$

$$\Rightarrow \angle APD + \angle CPD = 360^\circ - (\angle APD + \angle CPD)$$

$$\Rightarrow 2(\angle APD + \angle CPD) = 360^\circ$$

$$\Rightarrow \angle APD + \angle CPD = \frac{360}{2} = 180^\circ$$

This proves that CPA is a straight line.

31. Given LM is a line parallel to the Y-axis and its perpendicular distance from Y-axis is 3 units.

i. Coordinate of point P = (3,2)

Coordinate of point Q = (3,-1)

Coordinate of point R = (3, 0) [since its lies on X-axis, so its y coordinate is zero].

ii. Abscissa of point L = 3, abscissa of point M=3

\therefore Difference between the abscissa of the points L and M = 3 - 3 = 0

Section D

32. Given

$$a = 3 - 2\sqrt{2}$$

$$\Rightarrow a^2 = (3 - 2\sqrt{2})^2$$

$$= 3^2 - 2 \times 3 \times 2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 - 12\sqrt{2} + 8$$

$$= 17 - 12\sqrt{2}$$

$$\frac{1}{x^2} = \frac{1}{17 - 12\sqrt{2}}$$

$$= \frac{1}{17 - 12\sqrt{2}} \times \frac{17 + 12\sqrt{2}}{17 + 12\sqrt{2}}$$

$$= \frac{17 + 12\sqrt{2}}{17^2 - (12\sqrt{2})^2}$$

$$= \frac{17 + 12\sqrt{2}}{289 - 288}$$

$$= 17 + 12\sqrt{2}$$

$$\text{So } a^2 - \frac{1}{a^2} = (17 - 12\sqrt{2}) - (17 + 12\sqrt{2})$$

$$= 17 - 12\sqrt{2} - 17 - 12\sqrt{2}$$

$$= -24\sqrt{2}$$

OR

$$p = \frac{3 - \sqrt{5}}{3 + \sqrt{5}}$$

$$= \frac{3 - \sqrt{5}}{3 + \sqrt{5}} \times \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$$

$$= \frac{(3 - \sqrt{5})^2}{3^2 - (\sqrt{5})^2}$$

$$= \frac{9 + 5 - 6\sqrt{5}}{9 - 5}$$

$$= \frac{14 - 6\sqrt{5}}{4}$$

$$= \frac{7 - 3\sqrt{5}}{2}$$

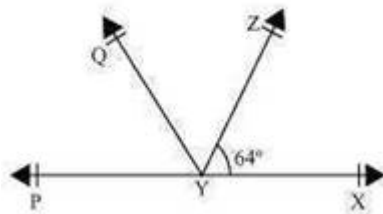
$$q = \frac{3 + \sqrt{5}}{3 - \sqrt{5}}$$

$$\begin{aligned}
&= \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} \\
&= \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2} \\
&= \frac{9+5+6\sqrt{5}}{9-5} \\
&= \frac{14+6\sqrt{5}}{4} \\
&= \frac{7+3\sqrt{5}}{2}
\end{aligned}$$

$$\begin{aligned}
p^2 + q^2 &= \left(\frac{7-3\sqrt{5}}{2}\right)^2 + \left(\frac{7+3\sqrt{5}}{2}\right)^2 \\
&= \frac{49+45-42\sqrt{5}}{4} + \frac{49+45+42\sqrt{5}}{4} \\
&= \frac{94-42\sqrt{5}}{4} + \frac{94+42\sqrt{5}}{4} \\
&= \frac{47-21\sqrt{5}}{2} + \frac{47+21\sqrt{5}}{2} \\
&= \frac{47-21\sqrt{5}+47+21\sqrt{5}}{2} \\
&= \frac{94}{2} \\
&= 47
\end{aligned}$$

- 33.
- Six points: A,B,C,D,E,F
 - Five line segments: $\overline{EG}, \overline{FH}, \overline{EF}, \overline{GH}, \overline{MN}$
 - Four rays: $\overrightarrow{EP}, \overrightarrow{GR}, \overrightarrow{GB}, \overrightarrow{HD}$
 - Four lines: $\overleftrightarrow{AB}, \overleftrightarrow{CD}, \overleftrightarrow{PQ}, \overleftrightarrow{RS}$
 - Four collinear points: M,E,G,B

34. We are given that $\angle XYZ = 64^\circ$, XY is produced to P and YQ bisects $\angle ZYP$ We can conclude the given below figure for the given situation:



We need to find $\angle XYQ$ and reflex $\angle QYP$

From the given figure, we can conclude that $\angle XYZ$ and $\angle ZYP$ form a linear pair.

We know that sum of the angles of a linear pair is 180° .

$$\angle XYZ + \angle ZYP = 180^\circ$$

$$\text{But } \angle XYZ = 64^\circ$$

$$\Rightarrow 64^\circ + \angle ZYP = 180^\circ$$

$$\Rightarrow \angle ZYP = 116^\circ$$

Ray YQ bisects $\angle ZYP$, or

$$\angle QYZ = \angle QYP = \frac{116^\circ}{2} = 58^\circ$$

$$\angle XYQ = \angle QYZ + \angle XYZ$$

$$= 58^\circ + 64^\circ = 122^\circ.$$

$$\text{Reflex } \angle QYP = 360^\circ - \angle QYP$$

$$= 360^\circ - 58^\circ$$

$$= 302^\circ.$$

Therefore, we can conclude that $\angle XYQ = 122^\circ$ and Reflex $\angle QYP = 302^\circ$

OR

PQ intersect RS at O

$$\therefore \angle QOS = \angle POR [\text{vert'ically opposite angles}]$$

$$a = 4b \dots(1)$$

Also,

$$a + b + 75^\circ = 180^\circ \text{ [} \because \text{POQ is a straight lines]}$$

$$\therefore a + b = 180^\circ - 75^\circ$$

$$= 105^\circ$$

Using, (1)

$$4b + b = 105^\circ$$

$$5b = 105^\circ$$

Or

$$b = \frac{105^\circ}{5} = 21^\circ$$

Now $a = 4b$

$$a = 4 \times 21^\circ$$

$$a = 84^\circ$$

Again, $\angle QOR$ and $\angle QOS$

$$\therefore a + 2c = 180^\circ$$

$$\text{Using, (2) } 84^\circ + 2c = 180^\circ$$

$$2c = 180^\circ - 84^\circ$$

$$2c = 96^\circ$$

$$c = \frac{96^\circ}{2} = 48^\circ$$

Hence,

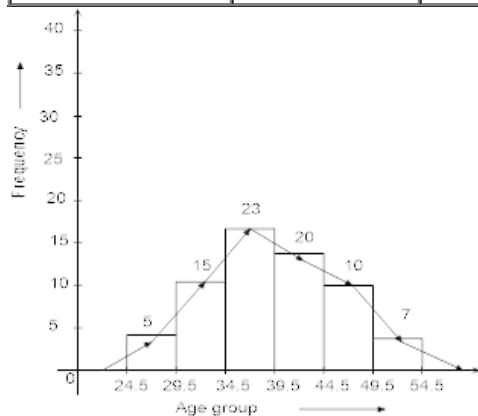
$$a = 84^\circ, b = 21^\circ \text{ and } c = 48^\circ$$

35. The given frequency distribution is not continuous. So we shall first convert it into a continuous frequency distribution.

The difference between the lower limit of a class and the upper limit of the preceding class is 1 i.e. $h=1$.

To convert the given frequency distribution into continuous frequency distribution, we subtract $\frac{h}{2}$ from lower limit and Add $\frac{h}{2}$ to upper limit $\therefore \frac{h}{2} = 0.5$ limit.

class interval	24.5 - 29.5	29.5 - 34.5	34.5 - 39.5	39.5 - 44.5	44.5 - 49.5	49.5 - 54.5
frequency	5	15	23	20	10	7



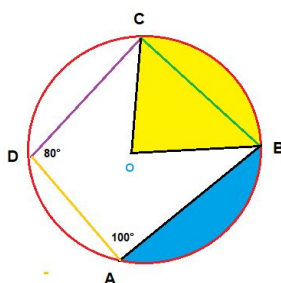
Section E

36. Read the text carefully and answer the questions:

There was a circular park in Defence colony at Delhi. For fencing purpose poles A, B, C and D were installed at the circumference of the park.

Ram tied wires From A to B, B to C and C to D, and he managed to measure the $\angle A = 100^\circ$ and $\angle D = 80^\circ$

Point O in the middle of the park is the center of the circle.



(i) ABCD is cyclic quadrilateral.

A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.

Here all four vertices A, B, C and D lie on a circle.

(ii) We know that the sum of both pair of opposite angles of a cyclic quadrilateral is 180° .

$$\angle C + \angle A = 1800$$

$$\angle C = 1800 - 1000 = 800$$

(iii) We know that

The sum of both pair of opposite angles of a cyclic quadrilateral is 180° .

$$\angle B + \angle D = 1800$$

$$\angle B = 1800 - 800 = 1000$$

OR

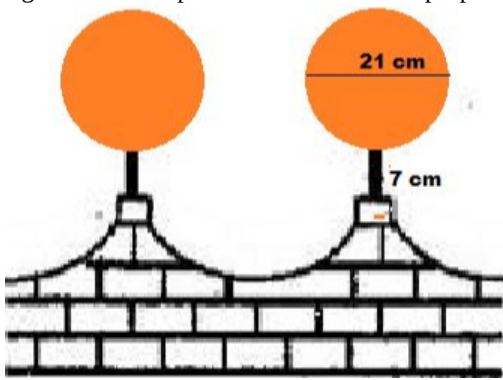
i. In a cyclic quadrilateral, all the four vertices of the quadrilateral lie on the circumference of the circle.

ii. The four sides of the inscribed quadrilateral are the four chords of the circle.

iii. The sum of a pair of opposite angles is 180° (supplementary). Let $\angle A$, $\angle B$, $\angle C$, and $\angle D$ be the four angles of an inscribed quadrilateral. Then, $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$.

37. Read the text carefully and answer the questions:

The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in figure. 25 such spheres are used for this purpose and are to be painted silver. Each support is a cylinder and is to be painted black.



(i) Diameter of a wooden sphere = 21 cm.
therefore Radius of wooden sphere (R) = $\frac{21}{2}$ cm

The surface area of 25 wooden spares

$$= 25 \times 4\pi R^2$$

$$= 25 \times 4 \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$$

$$= 138,600 \text{ cm}^2$$

(ii) Diameter of a wooden sphere = 21 cm.

therefore Radius of wooden sphere (R) = $\frac{21}{2}$ cm

The surface area of 25 wooden spares

$$= 25 \times 4\pi R^2$$

$$= 25 \times 4 \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$$

$$= 138,600 \text{ cm}^2$$

The cost of orange paint = 20 paise per cm^2

Thus total cost

$$= \frac{138600 \times 20}{100} = ₹ 27720$$

(iii) Radius of a wooden sphere $r = 4$ cm.

Height of support (h) = 7 cm

The surface area of 25 supports

$$= 25 \times \pi r^2 h$$

$$= 25 \times \frac{22}{7} \times 4^2 \times 7$$

$$= 8800 \text{ cm}^2$$

The cost of orange paint = 10 paise per cm^2

Thus total cost

$$= 0.1 \times 8800 = ₹ 880$$

OR

$$V = \frac{4}{3} \pi r^3 \times 25$$

$$V = 25 \times \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{6}\right)^3$$

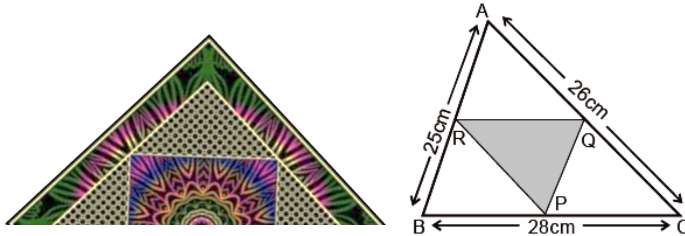
$$25 \times \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 25 \times 11 \times 21 \times 21$$

$$= 121275 \text{ cm}^3$$

38. Read the text carefully and answer the questions:

There is a Diwali celebration in the DPS school Janakpuri New Delhi. Girls are asked to prepare Rangoli in a triangular shape. They made a rangoli in the shape of triangle ABC. Dimensions of $\triangle ABC$ are 26 cm, 28 cm, 25 cm.



- (i) We know that line joining mid points of two sides of triangle is half and parallel to third side.

Hence RQ is parallel to BC and half of BC.

$$RQ = \frac{28}{2} = 14 \text{ cm}$$

Length of RQ = 14 cm

- (ii) By mid-point theorem we know that line joining mid points of two sides of triangle is half and parallel to third side.

$$PQ = \frac{AB}{2} = \frac{25}{2} = 12.5 \text{ cm}$$

$$QR = \frac{BC}{2} = \frac{28}{2} = 14 \text{ cm}$$

$$RP = \frac{AC}{2} = \frac{26}{2} = 13 \text{ cm}$$

$$\text{Length of garland} = PQ + QR + RP = 12.5 + 14 + 13 = 39.5 \text{ cm}$$

Length of garland = 39.5 cm..

- (iii) As R and P are mid-points of sides AB and BC of the triangle ABC, by mid point theorem, $RP \parallel AC$ Similarly, $RQ \parallel BC$ and $PQ \parallel AB$. Therefore ARPQ, BRQP and RQCP are all parallelograms. Now RQ is a diagonal of the parallelogram ARPQ, therefore, $\triangle ARQ \cong \triangle PQR$ Similarly $\triangle CPQ \cong \triangle RQP$ and $\triangle BPR \cong \triangle QRP$ So, all the four triangles are congruent.

Therefore Area of $\triangle ARQ = \text{Area of } \triangle CPQ = \text{Area of } \triangle BPR = \text{Area of } \triangle PQR$

Area $\triangle ABC = \text{Area of } \triangle ARQ + \text{Area of } \triangle CPQ + \text{Area of } \triangle BPR + \text{Area of } \triangle PQR$

Area of $\triangle ABC = 4 \text{ Area of } \triangle PQR$

$$\triangle PQR = \frac{1}{4} \text{ar}(\triangle ABC)$$

OR

As R and Q are mid-points of sides AB and AC of the triangle ABC. Similarly, P and Q are mid points of sides BC and AC by mid-point theorem, $RQ \parallel BC$ and $PQ \parallel AB$. Therefore BRQP is parallelogram